A New Simple and Accurate Formula for Microstrip Radial Stub

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***Abstract*— A new closed form formula is derived for the input impedance of the microstrip radial stub. The formula combines the advantages of simplicity of the radial transmission line approach and accuracy of the resonant mode expansion technique.**

I. Introduction

THE RADIAL STUB (RS) is in common use in both hybrid and monolithic microstrip circuits. When very low impedance levels are required, the behavior of the conventional stub degrades as a result of the excitation of higher order modes. The RS on the contrary provides a low impedance level at a well specified insertion point in a wide frequency band.

The presently available formulas for the input impedance of RS, however, are either poorly accurate or involve computation of slowly convergent series.

The rigorous modeling of the radial stub based on full-wave 3-D analysis would require excessive computer effort. For most applications simplified models based on a magnetic wall model [1] are of more practical use. A closed form formula for the input impedance of RS has been originally derived by Vinding [2] applying the theory for radial lines to a magnetic wall model of the stub. This model has been later modified in [3], [4] introducing an effective permittivity to account for the effects of fringing fields at the edges of the stub. Additional modifications of Vinding's formula have been proposed by March [5] to account for conductor and substrate loss.

Using the resonant mode expansion technique [1] that adopts a different effective permittivity depending on the field distribution of the resonant mode, a more accurate formula has been proposed in [6]. The penalty to be paid is that a slowly convergent series expansion must be used. In this letter, a new closed form formula is proposed that combines the simplicity of radial line approach and the accuracy of the resonant mode expansion. The accuracy of the new formula is demonstrated against the resonant mode expansion and experimental results.

II. Computation of Input Impedance

Fig. 1 shows a shunt connected microstrip RS, along with its effective geometry. This is used in the planar circuit approach to account for fringe field effects [1] and can be computed according to [6], [8]. The electromagnetic field in the RS

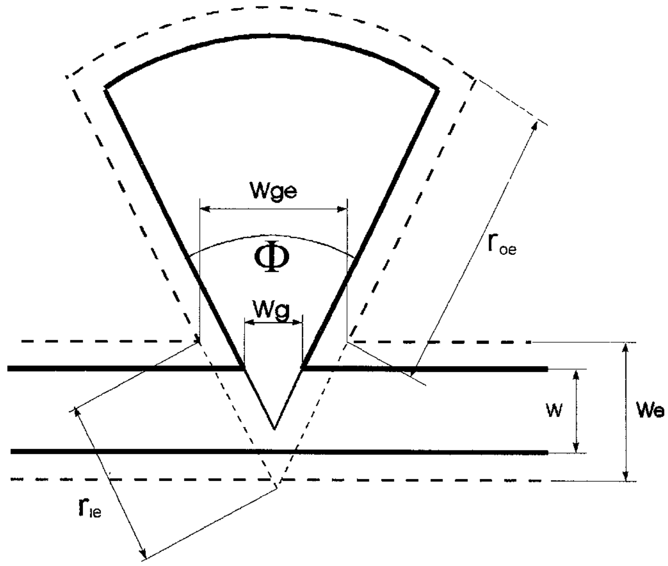

Fig. 1. Geometry of a shunt connected radial stub, the suffix e indicates effective dimension.

can be expressed in terms of radial line modes. Assuming that the input port width w is very small compared to the wavelength, only the dominant TEM radial mode is excited with nonnegligible amplitude. Accordingly, for an ideally lossless RS, the input impedance is found to be

$$Z_{\text{in}} = -jZ_0(r_{ie})\cot(kr_{ie}, kr_{oe}) \tag{1}$$

with

$$\cot(kr_{ie}, kr_{oe}) = \frac{N_0(kr_{ie})J_1(kr_{oe}) - J_0(kr_{ie})N_1(kr_{oe})}{J_1(kr_{ie})N_1(kr_{oe}) - N_1(kr_{ie})J_1(kr_{oe})}$$

$$Z_0(r_{ie}) = \frac{120\pi h}{r_{ie}\varphi\sqrt{\varepsilon_r}},$$

where the geometrical parameters of the RS are shown in Fig. 1, and Ji and Ni are the Bessel and Neumann functions of ith order.

This expression was presented by March [5] and is equivalent to that formerly derived by Vinding. In contrast with [5] however, effective parameters r_{ie}, etc., have been used in (1) to account for fringing fields. Nevertheless, the accuracy of (1) is still limited. A higher degree of accuracy can be obtained with the resonant mode expansion technique, where a different effective permittivity is ascribed to each resonant mode [1], [6]. In this manner, the input impedance of the RS

Manuscript received August 7, 1992.

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IEEE Log Number 9205149.

1051–8207/92$03.00

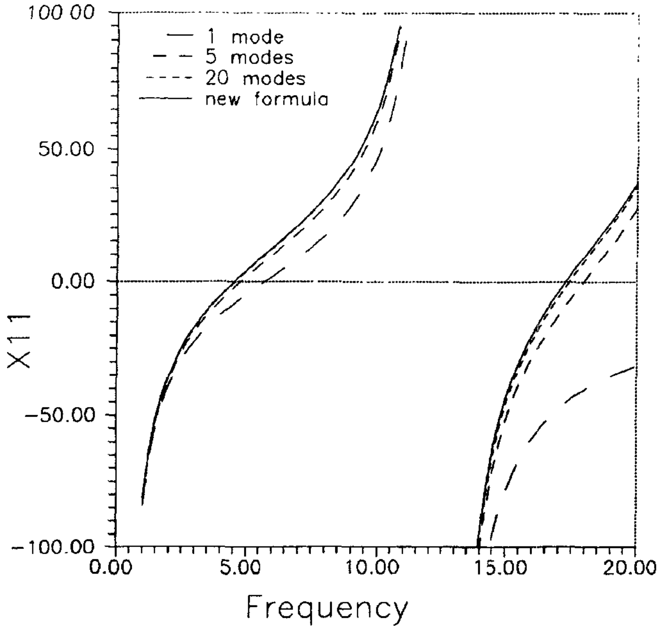

Fig. 2. Input reactance of a radial stub computed by the new formula (continuous line) and by the series expansion [6] (dashed lines). Stub parameters are: $Wg = 0.360$ mm; $\phi = 60°$, $r_o = 4.4$ mm, $\epsilon_r = 9.7$, substrate thickness $h = 0.635$ mm, metal thickness $t = 15$ μm.

is obtained in the form of a series expansion:

$$Z_{\rm in} = j\omega\mu h \frac{2}{\varphi} \sum_{n=1}^{\infty} \frac{P_n}{k_{0n}^2 - k^2 \varepsilon_{\text{eff},n}} + \frac{h}{j\omega\varepsilon_{\text{eff},0}} \cdot \frac{2/\varphi}{r_{oe}^2 - r_{ie}^2}, \quad (2)$$

where

$$P_n = \frac{C_0^2(k_{0n} r_{ie})}{[r^2 C_0^2(k_{0n} r)]_{r_{ie}}^{r_{oe}}}; \qquad P_0 = \frac{1}{r_{oe}^2 - r_{ie}^2}$$

$$C_0(x) = -\boldsymbol{J}_0(x) + Q\boldsymbol{N}_0(x); \qquad C_0(0) = 1$$

$$Q = -\boldsymbol{J}_1(k_{0n} r_{oe})/\boldsymbol{N}_1(k_{0n} r_{oe})$$

and the suffix e indicates the effective dimension.

It is apparent that although (2) has the advantage of higher accuracy, (1) has superior computational efficiency. We now observe that:

a) if the same ϵ_{eff} is assumed for all modes (including the static mode with $n = 0$), expression (2) converges exactly to (1);
b) the effective permittivities of all resonant modes have very close values, except when $n = 0$. This can be verified according to [10].

As a consequence, assuming $\epsilon_{\text{eff,n}} = \epsilon_{\text{eff},1}$ for any $n \geq 1$, (2) is easily transformed into a closed form expression:

$$Z_{\rm in} = -jZ_0(r_{ie}) \cot(kr_{ie}, kr_{oe}) - j\omega\mu h \frac{2}{\varphi} \frac{P_0}{k_{00}^2 - k^2 \varepsilon_{\text{eff},1}} + \frac{h}{j\omega\varepsilon_{\text{eff},0}} \cdot \frac{2/\varphi}{r_{oe}^2 - r_{ie}^2}. \quad (3)$$

This formula can be viewed as a correction to (1) to better account for the static fringing field capacitance. The corresponding reactance is represented by the third term in (3), while, as can easily be verified, the second term is the negative of the static term contained in the cot function.

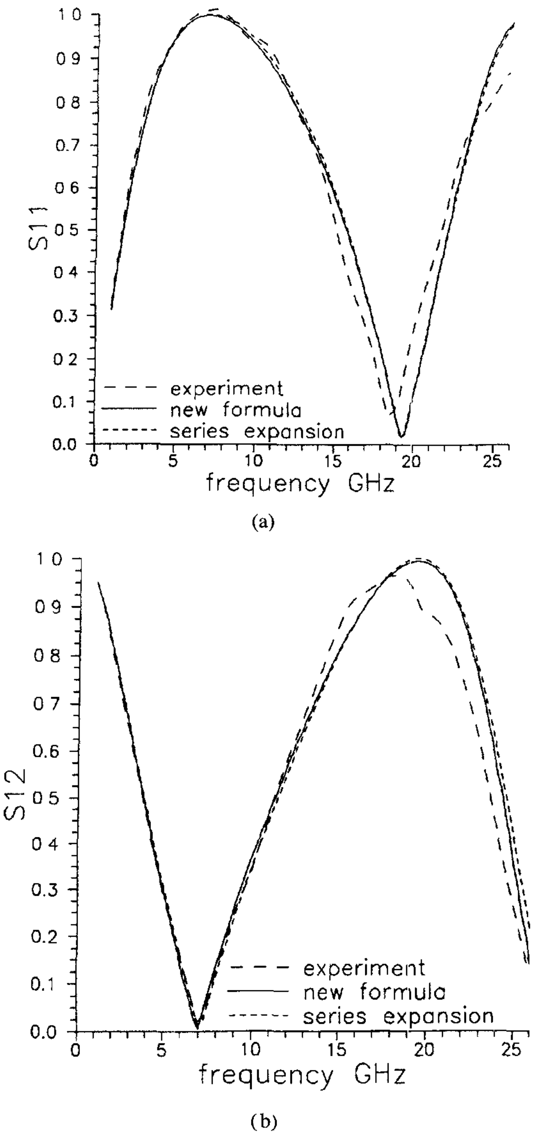

Fig. 3. Scattering parameters of a butterfly stub. Stub has the same parameters as in Fig. 2. Main line width $w = 0.642$ mm.

Conductor and dielectric losses can be accounted for in (3) by assuming a complex wavenumber: $k = \beta + j\alpha$ where

$$\alpha = \frac{R_s\sqrt{\varepsilon_r}}{120\pi} + \frac{\beta}{2}\tan(\delta)$$

$$R_s = \text{conductor surface resistivity in ohm/square.}$$

Once $Z_{\rm in}$ has been computed, the scattering parameters of shunt connected single or double (butterfly) stubs can be evaluated as in [6], i.e., considering the stub as a shunt load to the main line.

III. Results

The input impedance of a RS computed using both (2) and (3) are compared in Fig. 2. It is verified that the series expansion (2) converges to the new closed formula (3). At least 20 modal terms are required for convergence to be reached.

The accuracy of the new formula has been tested against several experimental data, as well as previously published results [6], [7]. An example is shown in Fig. 3 (a), (b), where the scattering parameters of a butterfly stub are plotted in the frequency range 2–26 GHz. The agreement is remarkable.

Acknowledgment

Mr. T. Sporkmann of ARGUMENS is gratefully acknowledged for providing the experimental results of radial stubs.

References

[1] R. Sorrentino, "Planar circuits, waveguide models, and segmentation method," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1057–1066, Oct. 1985.

[2] J. B. Vinding, "Radial line stubs as elements in stripline circuits," in *NEREM Records*, pp. 108–109, 1967.

[3] B. A. Syret, "A broad-band element for microstrip bias or tuning circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 925–927, Aug. 1980.

[4] H. A. Atwater, "Microstrip reactive circuit elements," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 488–491, June 1983.

[5] S. L. March, "Analyzing lossy radial-line stubs," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 269–271, Mar. 1985.

[6] F. Giannini, R. Sorrentino, and J. Vrba, "Planar circuit analysis of microstrip radial stub,"*IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1652–1655, Dec. 1984.

[7] F. Giannini, C. Paoloni, and M. Ruggieri, "CAD-oriented lossy models for radial stubs," *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 305–313, pp. 305–318, Feb. 1988.

[8] I. Wolff and N. Knoppik, "Rectangular and circular microstrip disk capacitors and resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 857–864, Oct. 1974.

[9] K. C. Gupta, R. Garg, and R. Chadha, *Computer-Aided Design of Microwave Circuits*. Dedham, MA: Artech House, 1981.

[10] J. Vrba, "Dynamic permittivity of microstrip ring resonator," *Electron. Lett.*, vol. 15, no. 16, pp. 504–505, Aug. 1979.